Decision by Sampling Implements Efficient Coding of Psychoeconomic Functions

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Abstract

The theory of decision by sampling (DbS) proposes that an attribute's subjective value is its rank within a sample of attribute values retrieved from memory. This can account for behavioral and neural data demonstrating context dependence beyond classic theories of decision making which assume stable preferences. In this paper, we provide a normative justification for DbS that is based on the principle of efficient coding. The efficient representation of information in a noiseless communication channel is characterized by a uniform response distribution, which the rank transformation implements. However, cognitive limitations imply that decision samples are finite, introducing noise. Efficient coding in a noisy channel requires smoothing of the signal, a principle that leads to a new generalization of DbS. This generalization helps descriptively account for a wider set of behavioral and neural observations, such as linearity in neural tuning curves, and variation in sensitivity to attribute range.

Keywords: decision making; efficient coding; smoothing

Decision by Sampling

Descriptive accounts of decision making such as expected utility theory are typically based on a stable set of "psychoeconomic" functions specifying the mental representations of gains, losses, probabilities, and delays. However, the psychological reality of such functions has been challenged by evidence that decisions are highly context-sensitive: the mental representation of an attribute changes depending on the choice set and other attribute values retrieved from memory. One influential account—***decision by sampling*** (DbS)—proposes that attributes of the current prospect are ordinally compared to attributes sampled from memory (Stewart, Chater, & Brown, 2006). By tallying these ordinal comparisons, a decision maker computes the value of a prospect's attribute as its rank relative to the distribution of attribute magnitudes in memory. Such nonlinear rank-based value representations have been observed in the brain (Mullett & Tunney, 2013). While DbS is a psychological process model, we show that the same set of ideas can be arrived at from a normative analysis based on the principle of efficient coding, which has a long history in the study of perceptual systems and has more recently been applied to neural representations of value (Louie & Glimcher, 2012).

The central contribution of our work is to clarify the computational design principles of DbS and related models, uniting them with an important strand of theoretical neuroscience. This paves the way for new behavioral predictions, insights into how DbS might be implemented in the brain, and a deeper understanding of the connections between information theory and decision making.

Efficient Coding and the Rank Transformation

According to the efficient coding principle, the brain is designed to communicate information in ways that minimize the costs of neural representation. This is accomplished by choosing a neural code that maximizes the mutual information between a neuron's inputs and outputs. When neurons are conceived as noiseless communication channels, maximizing mutual information is equivalent to minimizing redundancy, which can be achieved by recoding inputs according to their rank (Laughlin, 1981)—precisely the operation implemented by DbS in the limit of an infinite number of samples.

More formally, the mutual information between input (attribute) x and output (neural response) y is defined as $I(x;y) = H(y) - H(y|x)$, where $H(y)$ is the entropy of the output, and $H(y|x)$ is the conditional entropy of the output given the input. Noise in the channel is captured by $H(y|x)$ which reflects the residual uncertainty in the response knowing the stimulus. The principle of efficient coding as typically applied entails that stimuli should be encoded to maximize mutual information—that is, the mapping from x to y should maximize $I(x;y)$.

In the noiseless regime, $H(y|x)$ is 0, so maximizing mutual information is equivalent to maximizing output entropy (i.e., unpredictability). This is achieved by encoding x using the CDF, $y = F(x)$, also known as the probability integral transform, which guarantees that y is uniformly distributed. Since DbS approximates the probability integral transform, it can be understood as implementing efficient coding of psychoeconomic functions. In other words, DbS removes redundancies from the representations of gains, losses, probabilities, and delays, so that they can be represented with fewer bits (and thus presumably a lower metabolic cost). When the decision sample is large, the empirical rank $\hat{F}(x)$ will serve as a good approximation of the true rank $F(x)$.

Smoothing and Range Sensitivity

However, natural cognitive constraints imply that only a finite number of samples can be drawn from memory, in which case the channel becomes noisy. Efficiency can be partially restored by using a smoothed estimate of rank. Formally, one can heuristically satisfy the conflicting demands of redundancy reduction and information transmission by first smoothing the inputs prior to computing the probability integral transform, $\hat{F}_h(x;\mathbf{x}_{1:N}) = \frac{1}{N}\sum_{i=1}^{N} K\left(\frac{x-x_i}{h}\right)$ with a sample $\mathbf{x}_{1:N}$, where $K(z) = \int_{-\infty}^{z} k(z')dz'$ is an integrated kernel function and h is a bandwidth parameter. From a coding perspective, smoothing spreads out stimulus representations to better use the entire representational space, and it can reduce the variance of the rank estimate. This idea suggests that the principle of smoothing may guide the development and assessment of psychoeconomic models.

Smoothing increases the linearity of response functions, producing sensitivity to the range of the sample (rather than its skew), which addresses a known limitation of DbS. Smoothing may thus explain how efficient coding can be consistent with linear neural tuning curves that adapt to the range of values even when the attribute distribution is highly skewed (Rustichini, Conen, Cai, & Padoa-Schioppa, 2017). The smoothed representation formally corresponds to range-frequency theory (Parducci, 1995), according to which evaluation is a mixture of the attribute's rank and its position within the range of the contextual distribution.

Optimal Smoothing and Categorical Judgment

If range-based representations arise from smoothing, then the balance between rank and range sensitivity may be predicted by factors that affect the optimal level of smoothing. One such factor is the granularity of available responses. Within the confines of certain tasks, people must respond using a limited number of categories. Our theory speaks also to this setting.

Parducci and Wedell (1986) find that the number of available response categories influences the effects of skewness and the apparent weighting of range and rank elements. As the number of categories increases, the range component becomes more dominant and skewness has a diminished effect on judgment. This may be explained by optimal smoothing. When only a small number of response categories are available, evaluation does not need to be as precise. Response granularity naturally mitigates the effects of noise, including the harm caused by finite samples. Hence, smoothing (and thus range sensitivity) is less necessary with few categories.

Smoothing as Reduced Discriminability

As suggested by Stewart et al. (2006) and later elaborated by Brown and Matthews (2011), range-like effects can be captured by DbS if one assumes that experienced attribute values are not perfectly discriminable in memory. The efficient coding framework offers another perspective on this connection: imperfect discrimination may be a mechanism for reducing coding errors.

Kernel smoothing from a sampling perspective can present as reduced discriminability between retrieved items. Suppose that when an item is drawn from memory, some uncertainty is felt about its true location. This entails that items won't be completely distinguishable, and those nearer each other will be harder to distinguish. These are the assumptions imposed by reduced discriminability models. The coarse binary comparisons of DbS are then replaced with graded assessments of order to allow some tolerance. Rather than simply determining whether the target is greater than each sample value, the differences between the target and the samples are judged as significant to varying degrees based on the level of uncertainty. This smoothed comparison is exactly what a kernel encodes. Thus smoothing has a natural cognitive implementation based on uncertainty in retrieved values.

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